

A two-slit experiment which distinguishes between standard and Bohmian quantum mechanics

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In this investigation, we have suggested a special two-slit experiment which can distinguish between the standard and the Bohmian quantum mechanics. At the first step, we have shown that observable individual predictions obtained from these two theories are inconsistent for a special case. But, at the ensemble level, they are consistent as was expected. Then, as another special case and using selective detection, it is shown that an observable disagreement between the two theories can exist at the ensemble level of particles. This can encourage new efforts for finding other inconsistencies between the two theories, theoretically and experimentally.

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I. INTRODUCTION

Since the standard quantum mechanics (SQM) and the Bohmian quantum mechanics (BQM) have similar sets of equations, it seems that these two must be empirically equivalent, particularly at the ensemble level of particles. Bohm and his collaborators believed that their theory yields, in every conceivable experiment, the same statistical results as SQM [1-4]. Bohm, himself, in responding to the question of whether there is any new prediction by his theory, said (1986): “*Not the way it’s done. There are no new predictions because it is a new interpretation to the same theory*” [4]. In fact, when Bohm presented his theory in 1952, experiments could be done with an almost continuous beam of particles, but not with individual particles. Thus, Bohm cooked his theory in such a fashion that it would be impossible to distinguish his theory from SQM. For this reason, when Bell [5] talked about the empirical equivalence of the two theories, he was more cautious: “*It [the de Broglie-Bohm version of non-relativistic quantum mechanics] is experimentally equivalent to the usual version insofar as the latter is unambiguous*”. Thus the question arises as to whether there are phenomena which are well defined in one theory (due to the presence of path for particles) but ambiguous in the other one or whether there are experiments which have different observable results in the two theories? At first, it seems that the transition of a quantum system through a potential barrier provides a good case. For this case, there is no well defined transit time between the two ends of the barrier in SQM, because time is considered to be a parameter and not a dynamical variable having a corresponding Hermitian operator. For BQM, however, the passage of a particle between any two points is conceptually well defined. But, the recent work of Abolhasani and Golshani [6] indicates that it is not practically feasible to use this experiment to distinguish between these two theories. However, there are new efforts to show that BQM can have different predictions from SQM. For example, Neumaier [7] claimed that BQM contradicts SQM. But, Marchildon [8] has argued that this claim is unfounded. Very recently, Ghose [9] has claimed that by devising a new version of the two-slit experiment in which the wave packets of a pair of momentum correlated identical bosonic particles are simultaneously diffracted from the two slits, one can distinguish between the two theories. Although Ghose’s work [9] is also disputed by Marchildon [8], but Ghose still believes that his basic conclusions are right [10].

In this work, in parallel to Ghose’s conclusion in [9], we have studied BQM’s predictions about a two-slit experiment with a different particular source to show existence of disagreement between SQM and BQM for two special cases of the experiment.

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II. SPECIFICATIONS OF THE TWO-SLIT EXPERIMENT

We have considered the following two-slit experiment. A pair of identical non-relativistic particles (bosonic or fermionic) originate simultaneously from a point source. We assume that the intensity of the beam is so low that at a time we have only a single pair of particles passing through the slits. Since the direction of the emission of each particle can be considered to be random, we assume that the detection screen registers only those pairs of particles that reach it simultaneously. Then, the interference effects of single particles are eliminated on the screen. Furthermore, it is assumed that the detection process has no causal role in the phenomenon of interference [3]. In the coordinate system (x, y) , the centers of the two slits with width $2\sigma_0$ are located at $(0, \pm Y)$. We take the incident wave to be a plane wave of the form

$$\psi_{in}(x_1, y_1; x_2, y_2; t) = a e^{i[k_x(x_1+x_2)+k_y(y_1+y_2)]} e^{-iEt/\hbar}, \quad (1)$$

where a is a constant and $E = E_1 + E_2 = \hbar^2(k_x^2 + k_y^2)/m$ is the total energy of the two-particle system. We assume that, the source is located very far from the two-slit screen on the x -axis as compared with Y , so that the approximation $k_y \simeq 0$ is valid. For mathematical simplicity, we avoid slits with sharp edges which produce mathematical complexity of Fresnel diffraction, i.e., we assume that the slits have soft edges, so that the Gaussian wave packets are produced along the y -direction, and that the plane waves along the x -axis remain unchanged [3]. In fact, the one-particle wave function should be represented by Gaussian wave packets rather than plane or spherical waves, as utilized by Ghose [9] and Marchildon [8], respectively. We take the time of the formation of the Gaussian wave to be $t = 0$. Then, the emerging wave packets from the slits A and B are

$$\psi_A(x, y) = a(2\pi\sigma_0^2)^{-1/4} e^{-(y-Y)^2/4\sigma_0^2} e^{i[k_x x + k_y(y-Y)]}, \quad (2)$$

$$\psi_B(x, y) = a(2\pi\sigma_0^2)^{-1/4} e^{-(y+Y)^2/4\sigma_0^2} e^{i[k_x x - k_y(y+Y)]}, \quad (3)$$

where σ_0 is the half-width of each slit.

Now, for this two-particle system, the total wave function at the detection screen, at time t , is

$$\begin{aligned} \psi(x_1, y_1; x_2, y_2; t) = & N [\psi_A(x_1, y_1, t)\psi_B(x_2, y_2, t) + \psi_A(x_2, y_2, t)\psi_B(x_1, y_1, t) \\ & + \psi_A(x_1, y_1, t)\psi_A(x_2, y_2, t) + \psi_B(x_1, y_1, t)\psi_B(x_2, y_2, t)] \end{aligned} \quad (4)$$

with

$$\psi_A(x, y, t) = a(2\pi\sigma_t^2)^{-1/4} e^{-(y-Y-u_y t)^2/4\sigma_0\sigma_t} e^{i[k_x x + k_y(y-Y-u_y t/2) - E_x t/\hbar]}, \quad (5)$$

$$\psi_B(x, y, t) = a(2\pi\sigma_t^2)^{-1/4} e^{-(y+Y+u_y t)^2/4\sigma_0\sigma_t} e^{i[k_x x - k_y(y+Y+u_y t/2) - E_x t/\hbar]}, \quad (6)$$

where $N = 1/[2(1 + e^{(-Y^2/2\sigma_0^2)})]$ is a reparameterization constant,

$$\sigma_t = \sigma_0 \left(1 + \frac{i\hbar t}{2m\sigma_0^2}\right) \quad (7)$$

and

$$\begin{aligned} u_y &= \frac{\hbar k_y}{m}, \\ E_x &= \frac{1}{2} m u_x^2, \end{aligned} \quad (8)$$

where u_x and u_y are initial group velocities corresponding to each particle in the x and y -directions, respectively.

It is well known from SQM that the probability of simultaneous detection of the particles at $y_1 = Q_1$ and $y_2 = Q_2$, on the screen, located at $x_1 = x_2 = D$ at $t = D/u_x$, is equal to

$$P_{12}(Q_1, Q_2, t) = \int_{Q_1}^{Q_1+\Delta} dy_1 \int_{Q_2}^{Q_2+\Delta} dy_2 |\psi(x_1, y_1; x_2, y_2; t)|^2. \quad (9)$$

The parameter Δ , which is taken to be small, is a measure of the size of the detectors. We shall compare this prediction of SQM with that of BQM.

III. THE PREDICTIONS OF BQM FOR THE SUGGESTED EXPERIMENT

In BQM, the complete description of a system is given by specifying the location of its particles, in addition to the wave function which has the role of guiding the particles according to the following guidance condition:

$$\vec{x}_i(\vec{x}, t) = \frac{1}{m_i} \vec{\nabla}_i S(\vec{x}, t) = \frac{\hbar}{m_i} \text{Im} \left(\frac{\vec{\nabla}_i \psi(\vec{x}, t)}{\psi(\vec{x}, t)} \right), \quad (10)$$

where $S(\vec{x}, t)$ is the phase of total wave function which is written in polar form

$$\psi(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n; t) = R(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n; t) e^{iS(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n; t)/\hbar}. \quad (11)$$

Here, the speed of the particles 1 and 2 in the direction y is given, respectively, by

$$\dot{y}_1(x_1, y_1; x_2, y_2; t) = \frac{\hbar}{m} \text{Im} \frac{\partial_{y_1} \psi(x_1, y_1; x_2, y_2; t)}{\psi(x_1, y_1; x_2, y_2; t)}, \quad (12)$$

$$\dot{y}_2(x_1, y_1; x_2, y_2; t) = \frac{\hbar}{m} \text{Im} \frac{\partial_{y_2} \psi(x_1, y_1; x_2, y_2; t)}{\psi(x_1, y_1; x_2, y_2; t)}. \quad (13)$$

With the replacement of $\psi(x_1, y_1; x_2, y_2; t)$ from (4), we have

$$\begin{aligned} \dot{y}_1 = N \frac{\hbar}{m} \text{Im} \{ & \frac{1}{\psi} [[-2(y_1 - Y - u_y t)/4\sigma_0\sigma_t + ik_y] \psi_{A_1} \psi_{B_2} \\ & + [-2(y_1 + Y + u_y t)/4\sigma_0\sigma_t - ik_y] \psi_{A_2} \psi_{B_1} \\ & + [-2(y_1 - Y - u_y t)/4\sigma_0\sigma_t + ik_y] \psi_{A_1} \psi_{A_2} \\ & + [-2(y_1 + Y + u_y t)/4\sigma_0\sigma_t - ik_y] \psi_{B_1} \psi_{B_2}] \}, \end{aligned} \quad (14)$$

$$\begin{aligned} \dot{y}_2 = N \frac{\hbar}{m} \text{Im} \{ & \frac{1}{\psi} [[-2(y_2 + Y + u_y t)/4\sigma_0\sigma_t - ik_y] \psi_{A_1} \psi_{B_2} \\ & + [-2(y_2 - Y - u_y t)/4\sigma_0\sigma_t + ik_y] \psi_{A_2} \psi_{B_1} \\ & + [-2(y_2 - Y - u_y t)/4\sigma_0\sigma_t + ik_y] \psi_{A_1} \psi_{A_2} \\ & + [-2(y_2 + Y + u_y t)/4\sigma_0\sigma_t - ik_y] \psi_{B_1} \psi_{B_2}] \}. \end{aligned} \quad (15)$$

On the other hand, from (5) and (6) one can see that,

$$\begin{aligned} \psi_A(x_1, y_1, t) &= \psi_B(x_1, -y_1, t), \\ \psi_A(x_2, y_2, t) &= \psi_B(x_2, -y_2, t), \end{aligned} \quad (16)$$

which indicates the reflection symmetry of $\psi(x_1, y_1; x_2, y_2; t)$ with respect to the x -axis. Using this symmetry in (14) and (15), we have

$$\begin{aligned} \dot{y}_1(x_1, y_1, t) &= -\dot{y}_1(x_1, -y_1, t), \\ \dot{y}_2(x_2, y_2, t) &= -\dot{y}_2(x_2, -y_2, t). \end{aligned} \quad (17)$$

These relations show that, if $y_1(t) = 0$ or $y_2(t) = 0$, then the speed of each particles in the y -direction is zero along the symmetry axis x . This means that, none of the particles can cross the x -axis nor are tangent to it. The fact that the paths of the two particles are located on the two sides of the x -axis can lead, under suitable conditions, to a discrepancy between the predictions of SQM and BQM, particularly at the ensemble level.

If we consider $y = (y_1 + y_2)/2$ to be the vertical coordinate of the center of mass of the two particles, then we can write

$$\begin{aligned} \dot{y} &= (\dot{y}_1 + \dot{y}_2)/2 \\ &= N \frac{\hbar}{2m} \text{Im} \{ \frac{1}{\psi} [(-\frac{y_1 + y_2}{2\sigma_0\sigma_t}) (\psi_{A_1} \psi_{B_2} + \psi_{A_2} \psi_{B_1} + \psi_{A_1} \psi_{A_2} + \psi_{B_1} \psi_{B_2}) \\ &\quad + (\frac{Y + u_y t}{\sigma_0\sigma_t} + 2ik_y) (\psi_{A_1} \psi_{A_2} - \psi_{B_1} \psi_{B_2})] \} \\ &= \frac{(\hbar/2m\sigma_0^2)^2 t (y_1 + y_2)/2}{1 + (\hbar/2m\sigma_0^2)^2 t^2} + N \frac{\hbar}{2m} \text{Im} \{ \frac{1}{\psi} (\frac{Y + u_y t}{\sigma_0\sigma_t} + 2ik_y) (\psi_{A_1} \psi_{A_2} - \psi_{B_1} \psi_{B_2}) \}. \end{aligned} \quad (18)$$

Now, we consider the two following special cases:

A. The $\langle y_0 \rangle = 0$ and $\hbar t/2m\sigma_0^2 \sim 1$ condition

At first, consider the conditions $k_y \simeq 0$ and $Y \ll \sigma_0$. Then, we can consider

$$\psi_{A_1}\psi_{A_2} - \psi_{B_1}\psi_{B_2} \simeq 0, \quad (19)$$

in the second term of eq. (18). Hence, that equation of motion for the y -coordinate of the center of mass is converted to

$$\dot{y} \simeq \frac{(\hbar/2m\sigma_0^2)^2}{1 + (\hbar/2m\sigma_0^2)^2 t^2} y t. \quad (20)$$

Had we omitted the last two terms in the wave function (4), as was done in [9], we would have obtained the same result, exactly. Solving the differential equation (20), we get the path of the y -coordinate of the center of mass in the form

$$y \simeq y_0 \sqrt{1 + (\hbar/2m\sigma_0^2)^2 t^2}. \quad (21)$$

If the center of mass of the system was exactly on the x -axis at $t = 0$, i.e. $y_0 = 0$, then the center of mass of the system would have always remained on the x -axis and all the joint detections of the two particles would have happened symmetrically around the x -axis. However, based on the quantum equilibrium hypothesis, y_0 has an initial quantum distribution according to $|\psi|^2$ at $t = 0$. Thus, it may seem that we do not have symmetrical detection around the x -axis. On the other hand, it is well known that, the distance between any two neighboring maxima is nearly given by $\delta y \simeq \lambda x/2Y$, where λ is the de Broglie wavelength. If we consider δy as a length scale on the screen, then to obtain a symmetrical detection with a reasonable approximation, it would be enough that the center of mass deviation from the x -axis to be considered smaller than the distance between the neighboring maxima, that is,

$$\Delta y \ll \delta y \simeq \frac{\lambda D}{2Y} \simeq \frac{\pi \hbar t}{Y m}. \quad (22)$$

By considering the conditions $\hbar t/2m\sigma_0^2 \sim 1$ and $\Delta y_0 \sim \sigma_0$, one can obtain that the constraint of symmetrical detection of the two particles is

$$Y \ll 2\pi\sigma_0, \quad (23)$$

which is consistent with our applied initial assumption on Y and σ_0 in eq. (19). But, in SQM, there is a non-zero probability to find the two particles asymmetrical on the screen, as can be seen by eq. (9). Therefore, we have obtained a disagreement between the asymmetrical prediction of SQM and the symmetrical prediction of BQM for our special conditions.

It is worthy to note that, since the wave function specified in eq. (4) can be written in a factorizable form of wave function, namely

$$\psi(x_1, x_2; y_1, y_2; t) = N[\psi_A(x_1, y_1, t)\psi_B(x_2, y_2, t)][\psi_A(x_2, y_2, t)\psi_B(x_1, y_1, t)], \quad (24)$$

it can be seemed that the two particles behave independently at the ensemble level. Thus, SQM and BQM predict the same interference pattern for an ensemble pair of particles, as expected.

B. The $\langle y_0 \rangle \neq 0$ and $\hbar t/2m\sigma_0^2 \gg 1$ condition

In this case, we once again use eq. (21) for describing the time development of the center of mass y -coordinate, under the conditions $k_y \simeq 0$ and $Y \ll \sigma_0$. Furthermore, we try to perform our experiment in the following fashion: we record only those particles which are detected on the two sides of the x -axis, simultaneously. That is, we eliminate the cases of detecting only one particle or detecting the pairs which pass through the same slit, which means that we apply a selective detection on the particles. Thus, based on BQM, particularly the results obtained from (17), there will be a length

$$L \simeq 2\langle y \rangle \simeq \frac{\hbar t \langle y_0 \rangle}{m\sigma_0^2} \quad (25)$$

on the screen where almost no particle is recorded under the condition $\hbar t/2m\sigma_0^2 \gg 1$, if the constraint $\Delta y \ll L$ is satisfied. This constraint will turn into

$$\sigma_0 \ll \langle y_0 \rangle, \quad (26)$$

if the quantum equilibrium constraint $\Delta y_0 \sim \sigma_0$ is considered.

However, based on SQM, we have two alternatives:

- i) The probability relation (9) is still valid and due to the selective detection there is only a reduction in the intensity of the particles.
- ii) SQM is silent about our selective detection.

In the first case, there is a disagreement between predictions of SQM and BQM and in the second case, BQM shows a better predictive power than SQM, even at the ensemble level. Therefore, it seems that performing such experiment provides observable differences between the two theories, particularly at the ensemble level.

IV. CONCLUSION

We have shown that, a two-slit experiment, with a special source emitting two unentangled identical particles and with the condition $Y \ll \sigma_0$, can yield different predictions for SQM and BQM, in two special cases. In the case $\langle y_0 \rangle = 0$ and $\hbar t/2m\sigma_0^2 \sim 1$, BQM results symmetrical detection of the two particles around the x -axis with reasonable approximation, while according to SQM the probability of finding the two particles at arbitrary points on the screen is not zero. Furthermore, in the case $\langle y_0 \rangle \neq 0$ and $\hbar t/2m\sigma_0^2 \gg 1$, BQM predicts an empty interval on the screen if one uses selective detection, which is not predictable by SQM. Therefore, this experiment seems to shed light on the question of whether wave function provides a complete description of a system.

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